Idiosyncratic Heterogeneity and Aggregate Risk

Felix Wellschmied

UC3M

Macroeconomics III

The Idea

- We may want to study economies with heterogeneous agents AND aggregate risk.
- How income distribution changes over the business cycle.
- Distributional consequences of quantitative easing.
- Productivity consequences of job ladder.
- And and and ...

This May be "Easy"

- Kiyotaki Moore: Only two types.
- NKM: Price dispersion is irrelevant to first-order.
- Labor search: Perfect insurance.

Krusell-Smith Framework

It May be more "Difficult"

Household problem in Ayagari with aggregate risk:

$$\max_{c_t, k_{t+1}} \left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}$$

$$c_t + k_{t+1} = w_t \epsilon_t + k_t(1 + r_t)$$

$$k_{t+1} \geq \underline{k}$$

$$\pi^{\epsilon}_{jk}(\epsilon' = \epsilon^j | \epsilon = \epsilon^k)$$

$$\pi^{Z}_{jk}(Z' = Z^j | Z = Z^k)$$

$$w_t = Z_t(1 - \alpha) K_t^{\alpha} \bar{L}^{-\alpha}$$

$$r_t = Z_t \alpha K_t^{\alpha - 1} \bar{L}^{1-\alpha} - \delta$$

Recursive Equilibrium

1. Value function and policy functions that solve the household problem.
2. Markets clear: $K_t = \int k_i$, $L_t = \int \epsilon_i$.
3. Prices are given by $r_t = F_K(K_t, L_t) - \delta$, $w_t = F_L(K_t, L_t)$.
4. Law of motion for cross-sectional distribution $F_{t+1}(k_{t+1}, \epsilon_{t+1}, Z_{t+1}) = \Gamma(F_t)$.

Existence of Equilibrium

Miao (2006) show that a recursive equilibrium exists given the state variables:

1. Individual assets k_i.
2. Idiosyncratic shocks ϵ_i.
3. Aggregate shocks Z_t.
4. Cross sectional distribution $F_t(k_t, \epsilon_t, Z_t)$.
5. Cross sectional distribution of discounted utilities!

Uniqueness of Equilibrium

- No proof of uniqueness exists.
- My decisions depend on cross sectional distribution and implied policy of others.

Problem of Solving the Model

- Solution to the model:

$$u'(c_t) = \beta \mathbb{E}_t\{(1 + r_{t+1})u'(c_{t+1})\}$$
$$k_{t+1} + c_t = (1 + r_t)k_t + w_t\epsilon$$
$$K_t = \int k_i \Longrightarrow r_t, w_t$$

- Households need r_t, w_t and expectations about tomorrow to solve optimal k_t.
- In Aiyagari, we could solve the problem because $r_t = r_{t+1}, w_t = w_{t+1}$.
- With one time aggregate shock we could solve for finite transition path.
- With stochastic Z_t, this is not true.

What is the Problem? II

- Agents need $\mathbb{E}_t K_{t+1} = \int k^i_{t+1}$ to determine $\mathbb{E}_t r_{t+1}$.
- $\mathbb{E}_t K_{t+1}$ function of distribution of agents over assets. Depends on history of $Z_{s=0:t}$ and F_0.
- State space becomes $\Omega = k_t, \epsilon_t, Z_t, F_t(k, \epsilon, Z)$.
- This distribution is too complex numerically (infinite dimension).

Common Problem

- On the job random search: Firms decisions today depend on distribution of workers over firms.
- Investment: Firms need to know interest rate tomorrow, which results from individual firms' decisions today.
- Wealth heterogeneity in NKM.
- Wealth and labor supply over the business cycle.
- And, and, and ...

The Solution (Krusell and Smith (1998))

- Households use finite set of moments (m_t) from distribution predicting K_{t+1}.

- Test goodness of fit.

1. Guess law of motion for capital $\hat{K}_{t+1} = f(Z_t, m_t)$.
2. Solve individual household problem on space $\Omega_a = \epsilon_t, k_t, Z_t, m_t$.
3. Simulate an economy given individual policy rules. Note $\hat{K}_{t+1} \neq K_{t+1}$.
4. Update law of motion.
5. If law of motion not converged, go back to (2).

The Solution in Practice

- Usually, using first moment of distribution does good job.
- Approximate aggregation: Policies close to linear.
- Usually, linear regressions are used. R^2 as goodness of fit.
- Think about problem. *Log* often makes sense. Probit for probabilities...
- Interaction terms are possible, but multicollinearity is common.
- Extrapolation works only to some degree...

Value Function in Krusell-Smith

The transformed problem is:

$$V(k,\epsilon,Z,\bar{K}) = \max_{c,k'} \Big\{ U(c) + \beta \mathbb{E} V(k',\epsilon',Z',\bar{K}') \Big\}$$

$$c + k' = w\epsilon + k(1+r)$$

$$log(\bar{K}'(Z)) = \beta_0 + \beta_1 log(\bar{K}(Z)).$$

First order condition:

$$u'(c_t) = \beta E_t\{(1+r_{t+1})u'(c_{t+1})\}.$$

Justification for Krusell-Smith

- It is a numerical approximation to true model.

  As is everything else.

- Bounded rationality of agents.

  RE are not necessarily a good model.

  Without them, everything goes.

  Krusell-Smith puts bounds on what goes.

Khan and Thomas (2008)

The Idea

- Investment at the micro level is lumpy.

  More than half of investment occurs in one year.

  Time dependence.

- Large changes in investment demands

  increase each firms desire to invest.

  shrinks the hazard of investment.

- Households need to be willing to supply the funds.

  GE price effects dampen investment spikes.

- How important is lumpy investment for the business cycle?

The Set-up

- There is a representative household making consumption, savings, and labor supply decisions.
- The heterogeneity is on the firm side. They have heterogeneous productivities, ϵ_{it}, and investment costs, ξ_{it}.
- Apart of idiosyncratic productivity, firms also face stochastic aggregate productivity, z_t.
- Aggregate productivity growth deterministically at rate $\gamma - 1$.

Production and Costs

Firm produces output according to

$$Y = z\epsilon F(k, n) \quad Pr(z' = z_j | z = z_i) = \pi^z_{ij}$$
$$Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) = \pi^\epsilon_{ij}$$

Each period draw cost of investment (in wage units ω):

$$\xi \in [0, B] \sim G(\xi).$$

$i \neq 0,$	$\text{cost} = \omega\xi,$	$\gamma k' = (1-\delta)k + i$
$i = 0,$	$\text{cost} = 0,$	$\gamma k' = (1-\delta)k$

Aggregate state: (z, μ) with μ distribution of plants over k and ϵ.

Households

Households value consumption and leisure:

$$W(\lambda; z, \mu) = \max_{C,N,\lambda'} \{U(C, 1-N) + \beta \sum_{j=1}^{J} \pi_{ij} W(\lambda'; z_j, \mu')\}$$

$$C + \int \rho_1(k', \epsilon';, z, \mu)\lambda'(d[\epsilon' \otimes k'])$$

$$= \omega(z, \mu)N + \int \rho_0(k, \epsilon; z, \mu)\lambda(d[\epsilon \otimes k]).$$

with ρ being the price of shares in firms with capital stock k, and productivity ϵ.

The Firm Problem

$$v^1(k,\epsilon,\xi;z,\mu) = \max_{n,k^*}\Big\{z\epsilon F(k,n) - \omega(z,\mu)n + (1-\delta)k$$

$$+ \max\{-\xi\omega + r(k^*,\epsilon,\xi;z,\mu'), r((1-\delta)k,\epsilon,\xi;z,\mu')\}\Big\}$$

$$r(k',\epsilon,\xi;z,\mu') = -\gamma k' + \sum_{j=1}^{J}\pi_{ij}^{z} d_j(z,\mu)\sum_{m=1}^{M}\pi_{lm}^{\epsilon} v^0(k',\epsilon_m,z_j,\mu')$$

$$v^0(k,\epsilon,z,\mu) = \int_0^B v^1(k,\epsilon,\xi;z,\mu)G(d\xi)$$

where $d_j(z,\mu)$ is the stochastic discount factor of the firm used to value future dividends. Note, for notation, undepreciated capital is part of profits and firms buy back each period their capital stock.

Equilibrium requires

Let $p(z, \mu)$ be the price at which firms value current dividends. As firms are owned by the household, they value dividends at marginal utilities:

$$
\begin{aligned}
p(z, \mu) &= U_1(C, 1 - N) \\
d_j(z, \mu) &= \beta \frac{U_1(C', 1 - N')}{U_1(C, 1 - N)} \\
\omega(z, \mu) &= \frac{U_2(C, N - 1)}{U_1(C, N - 1)} = \frac{U_2(C, N - 1)}{p(z, \mu)}.
\end{aligned}
$$

Reformulating the Firm Problem

Write everything in terms of marginal utilities and note that n and k^* can be choosen independently:

$$v^1(k, \epsilon, \xi; z, \mu) = \max_n \Big\{ [z\epsilon F(k, n) - \omega(z, \mu)n + (1-\delta)k]p \Big\}$$
$$+ \max \Big\{ -\xi\omega p + \max_{k^*}\{R(k^*, \epsilon, \xi; z, \mu')\}, R((1-\delta)k, \epsilon, \xi; z, \mu') \Big\}$$

$$R(k', \epsilon, \xi; z, \mu') = -\gamma k' p + \beta \sum_{j=1}^{J} \pi^z_{ij} \sum_{m=1}^{M} \pi^\epsilon_{lm} V^0(k', \epsilon_m, z_j, \mu')$$

$$V^0(k, \epsilon, z, \mu) = \int_0^B V^1(k, \epsilon, \xi; z, \mu) G(d\xi).$$

Optimal Choices

Labor choice:

$$\omega(z,\mu) = z\epsilon F_2(k,n).$$

Capital choice:

$$-\xi\omega p + \max_{k^*}\{-\gamma k^* p + \beta \sum_{j=1}^{J} \pi_{ij}^{z} \sum_{m=1}^{M} \pi_{lm}^{\epsilon} V^0(k', \epsilon_m, z_j, \mu')\},$$

is independent of k. All adjusting plants choose $k^*(z,\epsilon,\mu)$.

$$k' = \begin{cases} k^*(z,\epsilon,\mu) & \text{if } \xi \leq \bar{\xi}(k,\epsilon;z,\mu) \\ (1-\delta)k & \text{if } \xi > \bar{\xi}(k,\epsilon;z,\mu). \end{cases}$$

Aggregate Dynamics

To solve the firm problem, we need to know $\mu' = \Gamma(z, \mu)$. and $p = \Lambda(z, \mu)$.

Replace μ by the mean capital stock.
For each productivity j, estimate:

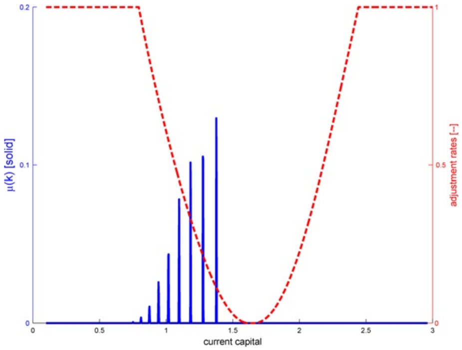
$$
\begin{aligned}
ln(\bar{K}') =& \beta_0^j + \beta_1^j \bar{K} \quad R_j^2 \approx 1 \\
ln(p) =& \gamma_0^j + \gamma_1^j \bar{K} \quad R_j^2 \approx 1.
\end{aligned}
$$

Calibration

- Match long run moments of US time series.
- Adjustment cost draws are uniformly distributed.

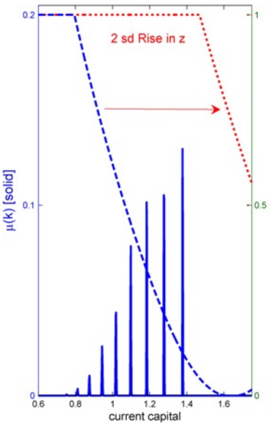
  Choose the upper bound to match lumpiness.
- Compare model to frictionless model.

Adjustment Hazard



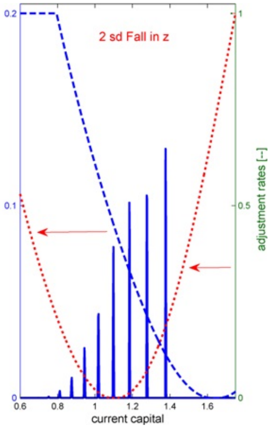
- Fix ϵ, z, μ.
- Minimum reached at $k^*(\epsilon, z, \mu)\frac{1-\delta}{\gamma}$.
- The further away, the more likely the firm becomes to adjust.

A Rise in Productivity (Fixed Prices)



- Adjustment hazard shifts to the right.
- Along the distribution more firms want to invest.

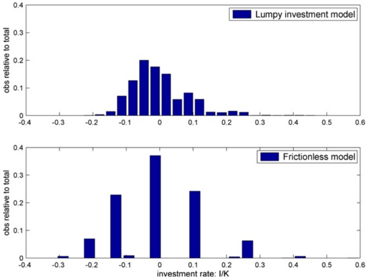
A Fall in Productivity (Fixed Prices)



- Adjustment hazard shifts to the left.

- Firms in the left of the distribution are less likely to adjust. Firms in the right, more likely.

Simulate Model with Fixed Prices



- Lumpy investment relative to reference:

  More time spend in fast growing investment.

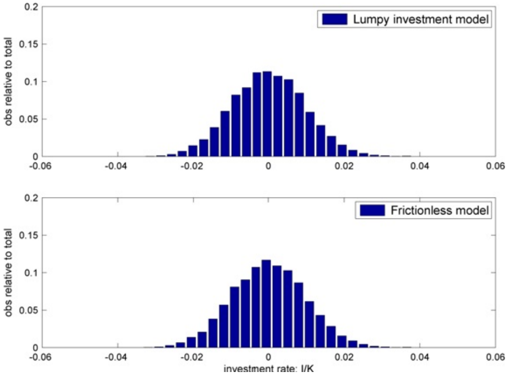
  Less in rapidly contracting.

General Equilibrium

	Output	TFP[a]	Hours	Consump.	Invest.	Capital
A. Standard deviations relative to output[b]						
GE frictionless	(2.277)	0.602	0.645	0.429	3.562	0.494
GE lumpy	(2.264)	0.605	0.639	0.433	3.539	0.492
B. Contemporaneous correlations with output						
GE frictionless		1.000	0.955	0.895	0.976	0.034
GE lumpy		1.000	0.956	0.900	0.976	0.034

- Business cycle basically identical to reference model.

General Equilibrium II



- Models feature much less volatility than partial equilibrium.
- Lumpy investment model almost identical to reference model.
- Despite the fraction of adjusting plants being strongly procyclical.

References

KHAN, A. AND J. K. THOMAS (2008): "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics," *Econometrica*, 76, 395–436.

KRUSELL, P. AND A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 105, 867–896.

MIAO, J. (2006): "Competitive equilibria of economies with a continuum of consumers and aggregate shocks," *Journal of Economic Theory*, 128, 274–298.